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ESTIMATES OF SECTORAL CAPITAL/OUTPUT  
RATIOS FOR KENYA

by

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ABSTRACT

Estimates of capital/output ratios for fourteen sectors of the Kenyan economy are reported. For each sector, five alternative estimates were calculated, each contingent on a different assumption about the rate of depreciation. Although the resulting estimates of capital/output ratios differ widely, it is shown that projections of gross investment requirements are quite insensitive to the choice of assumption. Related estimates of rates of profit on capital, and of the capital costs of providing jobs, are also discussed.

# Estimates of Sectoral Capital/Output Ratios for Kenya<sup>1</sup>

## 1. INTRODUCTION

Estimates of capital/output ratios are in great demand to help answer a variety of important questions in planning and policy. What are the investment requirements of growth at one rate or another? How rapid a growth rate is consistent with the capacity of the economy to save and to borrow abroad? What would be the imports of capital goods at various rates of growth? How do changes in the sectoral composition of GDP affect investment and import requirements? What are the investment costs of providing more jobs? And so on.

In the best of statistical circumstances, the capital/output ratio would not be the ideal concept to estimate and to apply to problems of planning and policy. It would be better to estimate complete production functions, embroidered with rates of technological progress and learning by doing. Unfortunately the Kenya data base is too short and too sparse, and the available time series are pervaded by a general upward trend that makes the inference of structural relations hazardous or impossible. A further difficulty is the changing composition of economic activities, even within sectors, in a developing country. Where growth is as much the accretion of new industries, activities, and technologies as the expansion of existing ones, the historical record may be a misleading guide.

The same problems limit the credence that can be given to any estimates of capital/output ratios or other simple summary indicators of production relationships. But they make less demand on the scarce supply of degrees of freedom, and they can at least provide a historical reference point for estimation of investment requirements. If planners and policy-makers have direct knowledge that, because of deliberate policy or changes in output mix or changes in technology, future investment will be more or less capital-intensive than past investment, they can use it to modify the statistical indications of historical experience.

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## 2. THE METHOD

For the reasons given in section 1 I have tried systematically to estimate capital/output ratios for fourteen sectors of the Kenyan economy. This has been done in the past, of course, but customarily just by computing the ratio of gross investment over a span of one or more years to the increment in GDP over the same period. The customary procedure is indicative, no doubt, but there are difficulties with it. No allowance is made for depreciation, and the implication is that no gross investment is needed if output is not growing. Or, to put the same objection in another way, the implication is that the gross investment needed depends solely on the absolute increment of output, regardless of the initial levels of output or capital. A statistical objection to the procedure is that the calculated ratios will naturally differ from year to year. We have no warrant to interpret these observed changes as genuine changes in the parameter we seek to measure, and no formal way to combine the various observations into a non-tautological estimate of the parameter. For this reason I have preferred to use the formal statistical technique of regression.

The basic data are the national accounts figures 1964-71 for GDP originating in a sector and for the sector's gross fixed capital formation, both in constant 1964 prices. Unfortunately I do not have a benchmark estimate of the stock of capital in any sector. Nor do I know the rates of depreciation in the various sectors. My procedure is to assume, for each sector, a number of alternative depreciation rates, specifically 0, .02, .04, .07, .10. For each depreciation rate a regression involving the two time series available, output and gross investment, yields an estimate of the capital/output ratio and also of the initial 1964 capital stock. There are then five estimates of this pair of parameters for each sector, one for each assumed rate of depreciation. They are shown in Table 1.

The model underlying the regression calculations is simply:

$$(1) \quad Y_t = \sigma K_t,$$

where:  $Y_t$  is GDP originating in the sector, at factor cost, in constant 1964 prices, during year  $t$ ;

$K_t$  is the stock of capital in the sector at the beginning of year  $t$ ;

$\sigma$  is the average and marginal productivity of the capital, assuming no shortage of labor and other cooperating factors. The capital/output ratio, which I denote  $\mu$ , is the reciprocal of  $\sigma$ .

(2)  $K_0 = K_0$ , the initial capital stock, not observed.

$$K_1 = K_0 s + I_0$$

$$K_2 = K_0 s^2 + I_0 s + I_1$$

.....

$$K_t = K_0 s^t + I_0 s^{t-1} + I_1 s^{t-2} + \dots + I_{t-1} \quad (t = 0, 7) \\ 1964 = 0.$$

Here  $s$  is the survival rate, equal to  $1-\delta$ , where  $\delta$  is the rate of depreciation. Thus  $s$  takes on the five arbitrary values 1, .98, .96, .93, .90.  $I_t$  is the observed gross investment at constant 1964 prices in year  $t$ .

Let  $\sum_{j=0}^{t-1} I_j s^{t-j-1}$  be  $V_t$ , so that  $K_t = K_0 s^t + V_t$ . For any assumed

survival rate  $s$ , the time series  $V_t$  can be calculated from the series  $I_t$ . The time series  $s^t$  can be calculated as soon as a value of  $s$  is assumed. Thus we can rewrite (1) as

$$(3) \quad Y_t = \sigma K_0 s^t + \sigma V_t = a s^t + b V_t$$

Regression of  $Y_t$  on  $s^t$  and  $V_t$ , with no constant term allowed in the regression, yields estimates  $a$  and  $b$ . The coefficient  $b$  is  $\sigma$  or  $1/\mu$ , the reciprocal of the capital/output ratio. The estimate of  $a$  is  $\sigma K_0$  - an estimate of  $Y_0$ , but not the actual observation - , so that  $K_0$  is estimated as  $a/b$ .

For projection of investment requirements, we wish to find  $I_t$  knowing  $Y_{t+1}$  and  $Y_t$ . If we assume that  $K_t = \mu Y_t$  ( $Y_t = \sigma K_t$ ), then  $K_{t+1}$  must be  $\mu Y_{t+1}$ , which is  $\mu(1+g)Y_t$  if  $g$  is the growth rate of  $Y$ . Thus the net investment requirement in year  $t$  is  $K_{t+1} - K_t = \mu g Y_t$ . To make up for depreciation further investment of  $\delta K_t = \delta \mu Y_t$  is required. This is the basis for the calculation in Table 2:  $I_t = \mu(g+\delta)Y_t$ .

One might hope to choose among the five pairs  $(\mu, \delta)$  for each sector by comparing the fits of the five regressions. Unfortunately this is almost always an empty criterion because all the fits are very good. In other words the data do not permit us to know whether they were generated by a process with depreciation rate zero or .10 or something in between. Choice, if any, must be made from external information. I will return to this problem below.

### 3. CALCULATING INVESTMENT REQUIREMENTS.

First, however, it is worth pointing out that for the calculation of investment requirements choice among the several estimates makes remarkably

little difference. The reason is that the capital/output ratios decline as the assumed depreciation rate increases. (See Table 1). When low depreciation rates are assumed large capital stocks are implied (Table 1, last five columns), and the regressions naturally report that they were needed to produce the observed outputs. When high depreciation rates are assumed, low capital stocks are implied, and the regressions report that the same observed outputs required little capital input. When the estimates thus obtained are used to calculate the gross investment required for a given sectoral GDP and its rate of growth, the low-depreciation estimate says there is a large net investment requirement but not much replacement investment, while the high-depreciation estimate says little net investment is required but a large amount of replacement.

This fact is illustrated in Table 2, where some hypothetical gross investment requirements for 1978 are estimated. The assumptions about sectoral rates of growth are quite arbitrary. In the table they are used both to project the levels of sectoral GDPs in 1978 and to describe the 1979/1978 growth in output for which investment in 1978 must provide. The sectors account for all of GDP except for the non-monetary sector, domestic service, and defense. A range of estimates of gross investment requirements can be obtained by summing (a) the lowest estimates for every sector, and (b) the highest estimates for every sector. The range is 123.6 to 136.8 Kenya pounds (1964 prices), .19 to .21 of the associated total GDP for the covered sectors. This compares with a figure of .23 actually observed in 1970. The example shows that the sectoral composition of output and growth does matter. In particular, sector P, Other Government Services, which includes roadbuilding, has a high capital/output ratio. In the period of observation the sector grew at nearly 16% per annum and by 1970 accounted for substantially more than half of general government capital formation. The illustrative halving of its growth rate, setting it at 8% per annum, equal to the average assumed for the whole economy, makes a big difference.

It is worth reminding readers and possible users of these numbers that the use of constant 1964 prices in the calculations is not as innocuous as it may seem. It assumes not only the general level of prices in 1964 - from which conversion to current prices could be made simply by applying a single GDP price index to the final figures - but also the structure of relative prices in 1964. In particular if the price of capital goods rises relative to the price of the output of the sector, the share of output required for gross investment will be higher than indicated in lines b and c in Table 2.

Those numbers would have to be multiplied by  $p_k/p_s$  where  $p_k$  is the expected price index (deflator) for capital formation in 1978 relative to 1964, and  $p_s$  is the expected index for the sectoral output. These corrections must be made sector by sector, and they might significantly modify the overall conclusions regarding the percentage of total GDP needed for investment. For example, there would be an upward revision if it were thought that imported capital goods will rise in price faster than domestic outputs.

The fact that disaggregation matters has already been pointed out. Further evidence of this is provided by comparing the aggregative estimates for the enterprise sector (A) as a whole with the sums of its nine constituent sectors. As indicated in Table 1, the estimates of initial 1964 capital stock for sector A are larger than the sum of the nine estimates B-J with the same uniform depreciation rate. The same exaggeration of capital requirements affects the calculation of gross investment requirements. When sector A is given the overall growth rate, 7.8% per year, implied by the assumed expansions of its constituent sectors in Table 2, the range of 1978 investment requirements for sector A is 112 to 116 million pounds. For the sum of sectors B-J, the range is 103 to 110 million pounds.

#### 4. LEVEL VERSUS FIRST DIFFERENCE REGRESSIONS.

In addition to the regressions of form (3) already reported, I computed regressions of the first differences of the variables:

$$(4) \quad \Delta Y_t = \sigma K_0 (s^t - s^{t-1}) + \sigma \Delta V_t = a(s^t - s^{t-1}) + b \Delta V_t$$

Unfortunately these turned out to be generally unsatisfactory. Estimates of  $a$  were frequently negative, a result that makes no economic sense in view of the model's identification of  $a$  with  $\sigma K_0$ . These estimates had large standard errors and were not significantly different from zero.

The estimates of  $b$  in (4), or in first difference regressions omitting  $(s^t - s^{t-1})$ , are always lower than the estimates from the level regressions (3). That is, the first difference regressions imply higher incremental capital/output ratios. A model which replaces (1) with:

$$(5) \quad Y_t = A + \sigma K_t,$$

would, assuming  $A$  positive, give a lower estimate of  $\sigma$ , i.e. a higher marginal capital/output ratio. It would imply a higher marginal than average ratio. But the further implication of (5) that production is possible without any capital is not appealing.



The difference regressions are more subject to errors in the independent variables, in particular to error in the identification of the constructed series for capital stock with the capital actually available and utilized in production. The regressions all assume that capital formed through the preceding year  $t-1$  produces the output of year  $t$ . Random deviations from the assumed timing make for relatively more substantial irregularities in the difference series  $\Delta V$  than in the level series  $V$ . The same is true of fluctuations in the rate of utilization of capital in existence.

As is well known, errors in independent variables bias regression coefficients toward zero, as well as magnifying their standard errors. For this reason I prefer the estimates of the level regressions, but one certainly cannot be very confident.

#### 5. CONSISTENCY WITH POWELL'S CAPITAL STOCK ESTIMATES.

Raymond Powell has painstakingly put together estimates of total fixed capital in Kenya 1964-71. His estimate for 1964 is 467 million pounds, for the monetary economy public and private. Inspection of Table 1 indicates that my estimates of initial capital stock would approximate this figure if the depreciation rate averaged to about .02.

#### 6. IMPLIED RATES OF PROFIT ON CAPITAL.

For some sectors it makes sense to compute the rates of profit (before tax) implied by the various estimates  $(\mu, \delta)$  in the hope that they may help to eliminate some pairs of estimates as implausible. Let  $m$  be the ratio of gross profits (including interest and other non-wage incomes) to gross value added in a sector at factor cost. Let  $p$  be the relative price correction,  $p_k/p_s$ , mentioned in section 3 above. Then an estimate of the rate of profit on capital at replacement cost is:

$$(6) \quad r = (m/\mu p) - \delta$$

This calculation does not make sense for sectors where a significant share of non-wage income is a return to investments in other than reproducible capital, land in the case of sector G, stocks in the case of sector H, financial equity in the case of sector I. Calculations for 1967, and in some cases for other years, are given in Table 3. The estimates are on the whole more plausible for the lower depreciation rates than for the higher.

## 7. CAPITAL COSTS OF JOBS.

A figure which interests many people is the amount of capital required to provide another job. This can be calculated as  $\mu y$  where  $y$  is GDP per man. Given the relative stability of  $\mu$  and the growth of  $y$ , capital per man increases in constant prices, roughly at the rate of growth of productivity. For anyone who would like to make such calculations, I provide in Table 4 some figures for GDP per man in 1970 (in 1964 prices), together with the trend growth of productivity over the years 1967-71.

Ideally a more relevant figure than  $\mu y$  would be  $(r+\delta)\mu y$  where  $r$  is the appropriate social discount rate. Allowance for depreciation rates could be important in comparing sectors and projects. Investments with high capital requirements per job should not be penalized if the investments are relatively very durable, nor projects with low capital per job favored if the capital involved wears out quickly.

For manufacturing, a capital/output ratio of about 2 means capital per job of about 1300 1964 pounds, or about 1600 1970 pounds. For electricity and water, a capital/output ratio of about 5 means capital per job of about 9000 pounds in 1970 prices. For all enterprises, using a capital/output ratio of 2.4 implies capital per job of nearly 2000 pounds in 1970 prices. Using Tables 1 and 4, the reader may make similar calculations for other sectors.

TABLE 1

RELATION OF ESTIMATED CAPITAL/OUTPUT RATIO TO ASSUMED DEPRECIATION RATE  
(Level Regressions 1964-71 data)

Sector	Estimated Capital/Output Ratio $\mu$ Assumption about Depreciation Rate $\delta$ :					Estimated Initial (1964) Capital Stock Assumption about Depreciation Rate $\delta$ :				
	$\delta = 0$	$\delta = .02$	$\delta = .04$	$\delta = .07$	$\delta = .10$	$\delta = 0$	$\delta = .02$	$\delta = .04$	$\delta = .07$	$\delta = .10$
Enterprises										
B. Manufacturing & Repair	2.45	1.98	1.66	1.34	1.12	81.83	66.53	56.03	45.43	38.29
C. Building & Construction	3.47	2.89	2.48	2.04	1.74	25.44	21.27	18.35	15.22	13.04
D. Elect. & Water	7.35	5.78	4.78	3.79	3.14	35.08	27.85	23.28	18.70	15.74
E. Transport & Communications	5.13	4.05	3.36	2.67	2.21	139.40	110.35	91.69	73.11	60.83
F. Mining & Quarrying	2.92	2.43	2.09	1.73	1.47	4.28	3.60	3.12	2.61	2.25
G. Agr., Forestry, Fishing	2.21	1.66	1.32	1.02	.83	115.53	86.91	69.74	53.85	43.96
H. Commerce	1.22	.94	.76	.59	.49	39.50	30.38	24.66	19.25	15.77
I. Banking, Ins., Real Estate	.41	.35	.31	.26	.21	4.03	3.47	3.04	2.57	1.99
J. Other Services, incl. Dwelling	3.11	2.55	2.16	1.77	1.49	39.52	32.85	28.19	23.40	20.05
A. All Enterprises	2.90	2.28	1.88	1.48	1.23	570.40	450.08	372.43	296.82	247.07
Sum B.J.						484.61	383.21	318.10	254.14	211.90
General Government										
K. Public Administration	1.43	1.05	.83	.63	.51	23.59	17.48	13.93	10.75	8.79
L. Education	.69	.59	.52	.44	.38	8.83	7.66	6.78	5.81	5.11
M. Health Services	1.77	1.45	1.22	1.00	.84	9.84	8.10	6.91	5.67	4.82
N. Agricultural Services	4.57	3.25	2.51	1.88	1.50	20.51	14.60	11.33	8.51	6.82
P. Other Government Services	5.03	4.55	4.15	3.68	3.30	17.35	15.82	14.54	13.01	11.78

TABLE 2

ILLUSTRATIVE CALCULATION OF INVESTMENT REQUIREMENTS 1978

Sector	1970 GDP (Millions 1964 pounds) Assumed growth rate p.a. 1970 (g)	Investment share of GDP= $\mu(g+\delta)$ Investment required (millions 1964 pounds)				
	1978 GDP (Millions 1964 pounds) (y) 1970 gross investment (millions 1964 pounds)	$\delta = 0$	$\mu(g+\delta)y$ .02	.04	.07	.10
B. Mfg, repair	52.49	.245	.238	.232	.228	.224
	.10					
	112.52	<u>27.57</u>	26.73	26.15	25.63	<u>25.20</u>
	12.31					
C. Bldg, constr.	12.07	.278	.289	.298	.306	.313
	.08					
	22.34	<u>6.20</u>	6.46	6.65	6.84	<u>7.00</u>
	6.55					
D. Elec., water	7.14	.588	.578	.574	.568	.565
	.08					
	13.22	<u>7.77</u>	7.64	7.58	7.52	<u>7.47</u>
	3.22					
E. Trans., comm.	41.18	.410	.405	.403	.400	.398
	.08					
	76.22	<u>31.28</u>	30.87	30.73	30.53	<u>30.32</u>
	18.47					
F. Mg, quarrying	2.60	.175	.194	.209	.225	.235
	.06					
	4.14	<u>.73</u>	.80	.86	.93	<u>.97</u>
	1.24					
G. Agr, forestry, fishing	75.35	.133	.133	.132	.133	.133
	.06					
	120.09	15.92	15.95	<u>15.85</u>	15.92	<u>15.95</u>
	11.10					
H. Commerce	48.64	.098	.094	.091	.088	.088
	.08					
	90.03	<u>8.79</u>	8.46	8.21	7.97	<u>7.94</u>
	3.84					
I. Banking, Insurance, Real Estate	19.40	.033	.035	.037	.039	.038
	.08					
	35.91	<u>1.18</u>	1.26	1.34	<u>1.40</u>	1.36
	1.53					
J. Other Services, incl. Dwellings	22.41	.218	.229	.238	.248	.253
	.07					
	38.50	<u>8.38</u>	8.84	9.15	9.54	<u>9.75</u>
	6.93					
K. Public Adm.	21.89	.086	.084	.083	.082	.082
	.06					
	34.89	<u>2.99</u>	2.93	2.90	2.86	2.85
	2.18					
L. Education	23.94	.069	.071	.073	.075	.076
	.10					
	51.32	<u>3.54</u>	3.63	3.74	3.84	<u>3.90</u>
	1.51					

Table 2 continued

M. Health Services	8.94	.177	.174	.171	.170	.168
	.10					
	19.16	<u>3.93</u>	3.33	3.27	3.26	<u>3.22</u>
	1.86					
N. Agr. Services	5.81	.457	.390	.351	.320	.300
	.10					
	12.45	<u>5.69</u>	4.85	4.38	3.98	<u>3.74</u>
	1.54					
P. Other Govt. services	9.42	.402	.455	.498	.552	.594
	.08					
	17.44	<u>7.02</u>	7.94	8.69	9.63	<u>10.35</u>
	9.51					
Total above sectors	351.28	Sum of <u>low</u> investment figures: 123.64				
	.08					
	648.23	Sum of <u>high</u> investment figures: 136.81				
	81.79	As shares of GDP: .191-.211				
		1970 investment share: .233				
Total sectors B-J	281.28	Sum of low investment figures: 103.27				
	.078					
	512.97	Sum of high investment figures: 110.48				
	65.19					
A. All enterprises	281.28	.226	.223	.222	.219	.219
	.078					
	512.97	<u>116.03</u>	114.60	113.78	112.34	<u>112.28</u>
	65.19					

TABLE 3

IMPLIED ESTIMATES OF RATES OF PROFIT ON CAPITAL

Sector and year	m Ratio of gross profits to value added	p Relative price correction	r = (m/μp)-δ -----Rate of profit-----					
			δ = 0	.02	.04	.07	.10	
B. Mfg., repair								
1964	.458	1.0	.17	.21	.23	.24	.27	
1965	.479	.991	.20	.22	.25	.29	.33	
1966	.488	.956	.21	.24	.27	.31	.36	
1967	.441	1.040	.17	.19	.21	.25	.28	
1968	.449	1.029	.18	.20	.22	.26	.29	
1969	.470	1.005	.19	.22	.24	.28	.32	
1970	.482	.934	.21	.24	.27	.32	.36	
C. Bldg, constr.								
1964	.116	1.0	.03	.02	.01	-.01	-.03	
1965	.213	1.005	.06	.05	.05	.03	.02	
1966	.241	.939	.07	.07	.06	.06	.05	
1967	.312	.919	.10	.10	.10	.10	.10	
1968	.470	.924	.15	.16	.16	.18	.19	
1969	.269	.910	.08	.08	.08	.07	.07	
1970	.226	.847	.08	.07	.07	.06	.05	
D. Elec, water								
1967	.679	1.000	.09	.10	.11	.13	.15	
1970	.730	1.079	.10	.11	.11	.12	.13	
E. Trans, comm								
1967	.380	1.235	.06	.06	.05	.05	.04	
F. Mng, quarrying								
1964	.411	1.000	.20	.22	.24	.27	.30	
1965	.404	1.038	.20	.22	.24	.26	.29	
1966	.484	1.067	.17	.18	.19	.21	.23	
1967	.500	1.107	.15	.17	.18	.19	.21	
1968	.497	1.121	.15	.16	.17	.19	.21	
1969	.545	1.170	.13	.14	.15	.15	.16	
1970	.511	1.273	.13	.14	.14	.15	.16	
J. Other Services								
1967	.610	1.133	.17	.19	.21	.23	.26	
A. All Enterprises								
1967	.534	1.125	.16	.19	.21	.25	.29	
1970	.482	1.078	.16	.19	.21	.25	.29	

TABLE 4

LABOR PRODUCTIVITY 1970 AND ITS RATE OF GROWTH 1967-71

Sector	GDP per man (thousands 1964 pounds)	rate of growth per year
Manufacturing and repair	.638	.017
Building and construction	.392	.043
Electricity and water	1.498	.107
Transport and communication	.917	.079
Mining and quarrying	.907	.020
Agr, forestry, fishing	.363	.034
Commerce	1.497	.053
Banking, insurance, real estate	2.002	.057
Other services	.539	.066
All Enterprises	.653	.044
General Government	.494	.041